

Proposition 9.1

We verify that the parametrization in Prop. 9.1 satisfies the equations in (32).

We record the equations and the parametrization. We evaluate the parametrized equations at the matrix M in (3).

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In[413]:= eqns = {u1 + u2 u5 u7 u13 u15, u10 + u5 u8 u9 u13 u14, u11 + u4 u5 u7 u8 u12 u13 u14 u15,
  u6 + u3 u7 u8 u12 u13, u9 + u4 u7 u10 u12 u15, u14 + u2 u3 u7 u10 u11 u12 u13 u15,
  u8 + u2 u6 u10 u11 u15, u4 + u2 u3 u9 u11 u13, u12 + u2 u5 u6 u9 u11 u13 u14 u15,
  u3 + u4 u5 u6 u14 u15, u2 + u1 u4 u8 u12 u14, u1 u3 u8 u9 u11 u12 u13 u14 + u15,
  u7 + u1 u6 u9 u11 u14, u5 + u1 u3 u10 u11 u12, u13 + u1 u4 u6 u10 u11 u12 u14 u15};
parametrization = {u1 -> (p135 * p146 * p234 * p256 - p134 * p156 * p235 * p246) /
  (p126 p135 p234 p456),
  u2 -> (p134 p156 p235 p246) / (p135 p146 p234 p256), u3 -> (p134 p356) / (p135 p346), u4 -> (p136 p145) / (p135 p146),
  u5 -> (p125 p136 p246 p345) / (p126 p135 p245 p346), u6 -> (p136 p235) / (p135 p236), u7 -> (p123 p145 p246 p356) / (p124 p135 p236 p456),
  u8 -> (p125 p356) / (p135 p256), u9 -> (p125 p134) / (p124 p135), u10 -> (p145 p235) / (p135 p245), u11 -> (p135 p234) / (p134 p235), u12 -> (p135 p456) / (p145 p356),
  u13 -> (p124 p135 p256 p346) / (p125 p134 p246 p356), u14 -> (p126 p135) / (p125 p136), u15 -> (p135 p146 p236 p245) / (p136 p145 p235 p246)};
PlueckerVar = {p123, p124, p125, p126, p134, p135, p136, p145, p146,
  p156, p234, p235, p236, p245, p246, p256, p345, p346, p356, p456};
dMatrix = {{1, 1, 1, 1, 1, 1}, {d1, d2, d3, d4, d5, d6}, {d1^3, d2^3, d3^3, d4^3, d5^3, d6^3}};
MatrixForm[dMatrix]
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Out[417]/MatrixForm=

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ d_1 & d_2 & d_3 & d_4 & d_5 & d_6 \\ d_1^3 & d_2^3 & d_3^3 & d_4^3 & d_5^3 & d_6^3 \end{pmatrix}$$

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In[404]:= Pluecker[mat_, id_] := Det[mat[[All, id]]];
allPluecker[mat_] := Map[Pluecker[mat, #] &, Subsets[{1, 2, 3, 4, 5, 6}, {3}]];
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Every equation in (32) evaluates to one.

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In[412]:= Simplify[(eqns /. parametrization) /. Thread[PlueckerVar -> allPluecker[dMatrix]]]
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Out[412]= {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}