

## Proof of Prop. 10.1

Here we verify the claim in the proof of Proposition 10.1 that, for each polygon on the cubic surface, there exist six pairwise disjoint lines that are disjoint from the polygon.

The following list of polygons are taken from the Section 3.

In[392]= truePolygons =

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Sort[{{"E4", "F24", "G2"}, {"E4", "F34", "G3"}, {"E5", "F15", "G1"},
{"E5", "F25", "G2"}, {"E6", "F16", "G1"}, {"E6", "F36", "G3"},
{"F14", "F25", "F36"}, {"F14", "F26", "F35"}, {"F15", "F26", "F34"},
{"F16", "F24", "F35"}, {"E1", "E5", "G3", "G4"},
{"E1", "E5", "G3", "G6"}, {"E1", "E6", "F16", "G5"},
{"E1", "E6", "G2", "G4"}, {"E1", "E6", "G2", "G5"},
{"E1", "E6", "G3", "G4"}, {"E1", "F12", "F14", "F36"},
{"E1", "F12", "F14", "G1"}, {"E1", "F12", "F15", "F36"},
{"E1", "F13", "F14", "G1"}, {"E1", "F13", "F16", "F25"},
{"E1", "F15", "F36", "G6"}, {"E1", "F16", "F25", "G5"},
{"E2", "E4", "G1", "G5"}, {"E2", "E4", "G1", "G6"},
{"E2", "E4", "G3", "G6"}, {"E2", "E5", "F25", "G4"},
{"E2", "E5", "G1", "G6"}, {"E2", "E5", "G3", "G4"},
{"E2", "E5", "G3", "G6"}, {"E2", "F12", "F25", "F34"},
{"E2", "F12", "F26", "F34"}, {"E2", "F12", "F26", "G2"},
{"E2", "F15", "F23", "F24"}, {"E2", "F15", "F24", "G5"},
{"E2", "F23", "F26", "G2"}, {"E2", "F25", "F34", "G4"},
{"E3", "E4", "G1", "G5"}, {"E3", "E4", "G1", "G6"},
{"E3", "E4", "G2", "G5"}, {"E3", "E6", "G2", "G4"},
{"E3", "E6", "G2", "G5"}, {"E3", "F13", "F24", "F36"},
{"E3", "F13", "F35", "G3"}, {"E3", "F16", "F23", "F34"},
{"E3", "F16", "F23", "F35"}, {"E3", "F16", "F34", "G6"},
{"E3", "F23", "F35", "G3"}, {"E3", "F24", "F36", "G4"},
{"E4", "F14", "F23", "F45"}, {"E4", "F14", "F23", "F46"},
{"E4", "F14", "F45", "G4"}, {"E4", "F14", "F46", "G4"},
{"E4", "F24", "F45", "G4"}, {"E4", "F34", "F46", "G4"},
{"E5", "F12", "F35", "F45"}, {"E5", "F12", "F35", "F56"},
{"E5", "F12", "F45", "G2"}, {"E5", "F15", "F56", "G5"},
{"E5", "F35", "F45", "G5"}, {"E5", "F35", "F56", "G5"},
{"E6", "F13", "F26", "F46"}, {"E6", "F13", "F26", "F56"},
{"E6", "F13", "F56", "G1"}, {"E6", "F26", "F46", "G6"},
{"E6", "F26", "F56", "G6"}, {"E6", "F36", "F46", "G6"},
{"F12", "F14", "F56", "G1"}, {"F12", "F15", "F36", "F46"},
{"F12", "F25", "F34", "F46"}, {"F12", "F26", "F35", "F45"},
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{"F12", "F26", "F45", "G2"}, {"F13", "F14", "F26", "F56"},
{"F13", "F14", "F56", "G1"}, {"F13", "F16", "F24", "F45"},
{"F13", "F16", "F25", "F45"}, {"F13", "F24", "F36", "F45"},
{"F13", "F25", "F36", "F45"}, {"F13", "F35", "F46", "G3"},
{"F14", "F23", "F35", "F46"}, {"F14", "F25", "F46", "G4"},
{"F15", "F23", "F24", "F56"}, {"F15", "F23", "F34", "F56"},
{"F15", "F24", "F56", "G5"}, {"F15", "F26", "F46", "G6"},
{"F15", "F36", "F46", "G6"}, {"F16", "F23", "F34", "F56"},
{"F16", "F25", "F45", "G5"}, {"F16", "F34", "F56", "G6"},
{"F23", "F26", "F45", "G2"}, {"F23", "F35", "F46", "G3"},
{"F24", "F35", "F56", "G5"}, {"F24", "F36", "F45", "G4"},
{"F25", "F34", "F46", "G4"}, {"E1", "E5", "F12", "F15", "G1"},
{"E1", "E5", "F15", "G1", "G6"}, {"E1", "E5", "F25", "G2", "G4"},
{"E1", "F25", "G2", "G5"}, {"E1", "E6", "F13", "F16", "G1"},
{"E1", "F36", "G3", "G6"}, {"E1", "F13", "F14", "F25", "F36"},
{"E2", "E4", "F23", "F24", "G2"}, {"E2", "E4", "F24", "G2", "G5"},
{"E2", "F34", "G3", "G4"}, {"E2", "E5", "F12", "F25", "G2"},
{"E2", "F15", "G1", "G5"}, {"E2", "F15", "F23", "F26", "F34"},
{"E3", "E4", "F23", "F34", "G3"}, {"E3", "F24", "G2", "G4"},
{"E3", "E4", "F34", "G3", "G6"}, {"E3", "E6", "F13", "F36", "G3"},
{"E3", "E6", "F16", "G1", "G5"}, {"E3", "F16", "G1", "G6"},
{"E3", "E6", "F36", "G3", "G4"}, {"E3", "F13", "F16", "F24", "F35"},
{"E4", "F23", "F24", "F45", "G2"}, {"E4", "F23", "F34", "F46", "G3"},
{"E5", "F12", "F15", "F56", "G1"}, {"E5", "F25", "F45", "G2", "G5"},
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{"F12", "F14", "F26", "F35", "F56"}, {"F12", "F15", "F26",
  "F34", "F46"}, {"F13", "F14", "F26", "F35", "F46"},
{"F14", "F23", "F26", "F35", "F45"}, {"F14", "F25", "F36", "F45", "G4"},
{"F15", "F26", "F34", "F56", "G6"}, {"F16", "F23", "F24", "F35", "F56"},
{"F16", "F24", "F35", "F45", "G5"}];

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We generate 30 groups of six pairwise disjoint lines and verify that for each polygon from the list above, one of these 30 groups works.

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In[ ]:= disjointSix = {"F23", "F24", "F25", "F26", "E1", "G1"},
  {"F23", "F34", "F35", "F36", "E1", "G1"},
  {"F24", "F34", "F45", "F46", "E1", "G1"}, {"F25", "F35", "F45",
  "F56", "E1", "G1"}, {"F26", "F36", "F46", "F56", "E1", "G1"},
  {"F13", "F14", "F15", "F16", "E2", "G2"}, {"F13", "F34", "F35",
  "F36", "E2", "G2"}, {"F14", "F34", "F45", "F46", "E2", "G2"},
  {"F15", "F35", "F45", "F56", "E2", "G2"}, {"F16", "F36", "F46",
  "F56", "E2", "G2"}, {"F12", "F14", "F15", "F16", "E3", "G3"},
  {"F12", "F24", "F25", "F26", "E3", "G3"}, {"F14", "F24", "F45",
  "F46", "E3", "G3"}, {"F15", "F25", "F45", "F56", "E3", "G3"},
  {"F16", "F26", "F46", "F56", "E3", "G3"}, {"F12", "F13", "F15",
  "F16", "E4", "G4"}, {"F12", "F23", "F25", "F26", "E4", "G4"},
  {"F13", "F23", "F35", "F36", "E4", "G4"}, {"F15", "F25", "F35",
  "F56", "E4", "G4"}, {"F16", "F26", "F36", "F56", "E4", "G4"},
  {"F12", "F13", "F14", "F16", "E5", "G5"}, {"F12", "F23", "F24",
  "F26", "E5", "G5"}, {"F13", "F23", "F34", "F36", "E5", "G5"},
  {"F14", "F24", "F34", "F46", "E5", "G5"}, {"F16", "F26", "F36",
  "F46", "E5", "G5"}, {"F12", "F13", "F14", "F15", "E6", "G6"},
  {"F12", "F23", "F24", "F25", "E6", "G6"}, {"F13", "F23", "F34",
  "F35", "E6", "G6"}, {"F14", "F24", "F34", "F45", "E6", "G6"},
  {"F15", "F25", "F35", "F45", "E6", "G6"}];

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In the output, the first set is the polygon and the second is the six pairwise disjoint lines.

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In[ ]:= For[i = 1, i < 131, i++,
  j = 1;
  While[Not[Intersection[truePolygons[[i]], disjointSix[[j]]] == {}], j++
  ];
  Print["Polygon=", truePolygons[[i]], " Lines=", disjointSix[[j]]
  ]
]
Polygon={E4, F24, G2} Lines={F23, F34, F35, F36, E1, G1}
Polygon={E4, F34, G3} Lines={F23, F24, F25, F26, E1, G1}
Polygon={E5, F15, G1} Lines={F13, F34, F35, F36, E2, G2}
Polygon={E5, F25, G2} Lines={F23, F34, F35, F36, E1, G1}
Polygon={E6, F16, G1} Lines={F13, F34, F35, F36, E2, G2}
Polygon={E6, F36, G3} Lines={F23, F24, F25, F26, E1, G1}
Polygon={F14, F25, F36} Lines={F24, F34, F45, F46, E1, G1}
Polygon={F14, F26, F35} Lines={F24, F34, F45, F46, E1, G1}
Polygon={F15, F26, F34} Lines={F25, F35, F45, F56, E1, G1}
Polygon={F16, F24, F35} Lines={F26, F36, F46, F56, E1, G1}
Polygon={E1, E5, G3, G4} Lines={F13, F14, F15, F16, E2, G2}

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Polygon={E1, E5, G3, G6} Lines={F13, F14, F15, F16, E2, G2}  
 Polygon={E1, E6, F16, G5} Lines={F13, F34, F35, F36, E2, G2}  
 Polygon={E1, E6, G2, G4} Lines={F12, F14, F15, F16, E3, G3}  
 Polygon={E1, E6, G2, G5} Lines={F12, F14, F15, F16, E3, G3}  
 Polygon={E1, E6, G3, G4} Lines={F13, F14, F15, F16, E2, G2}  
 Polygon={E1, F12, F14, F36} Lines={F15, F35, F45, F56, E2, G2}  
 Polygon={E1, F12, F14, G1} Lines={F13, F34, F35, F36, E2, G2}  
 Polygon={E1, F12, F15, F36} Lines={F14, F34, F45, F46, E2, G2}  
 Polygon={E1, F13, F14, G1} Lines={F15, F35, F45, F56, E2, G2}  
 Polygon={E1, F13, F16, F25} Lines={F14, F34, F45, F46, E2, G2}  
 Polygon={E1, F15, F36, G6} Lines={F14, F34, F45, F46, E2, G2}  
 Polygon={E1, F16, F25, G5} Lines={F13, F34, F35, F36, E2, G2}  
 Polygon={E1, F25, G2, G5} Lines={F12, F14, F15, F16, E3, G3}  
 Polygon={E1, F36, G3, G6} Lines={F13, F14, F15, F16, E2, G2}  
 Polygon={E2, E4, G1, G5} Lines={F12, F14, F15, F16, E3, G3}  
 Polygon={E2, E4, G1, G6} Lines={F12, F14, F15, F16, E3, G3}  
 Polygon={E2, E4, G3, G6} Lines={F23, F24, F25, F26, E1, G1}  
 Polygon={E2, E5, F25, G4} Lines={F23, F34, F35, F36, E1, G1}  
 Polygon={E2, E5, G1, G6} Lines={F12, F14, F15, F16, E3, G3}  
 Polygon={E2, E5, G3, G4} Lines={F23, F24, F25, F26, E1, G1}  
 Polygon={E2, E5, G3, G6} Lines={F23, F24, F25, F26, E1, G1}  
 Polygon={E2, F12, F25, F34} Lines={F26, F36, F46, F56, E1, G1}  
 Polygon={E2, F12, F26, F34} Lines={F25, F35, F45, F56, E1, G1}  
 Polygon={E2, F12, F26, G2} Lines={F23, F34, F35, F36, E1, G1}  
 Polygon={E2, F15, F23, F24} Lines={F25, F35, F45, F56, E1, G1}  
 Polygon={E2, F15, F24, G5} Lines={F23, F34, F35, F36, E1, G1}  
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 Polygon={E2, F23, F26, G2} Lines={F24, F34, F45, F46, E1, G1}  
 Polygon={E2, F25, F34, G4} Lines={F26, F36, F46, F56, E1, G1}  
 Polygon={E2, F34, G3, G4} Lines={F23, F24, F25, F26, E1, G1}  
 Polygon={E3, E4, G1, G5} Lines={F13, F14, F15, F16, E2, G2}  
 Polygon={E3, E4, G1, G6} Lines={F13, F14, F15, F16, E2, G2}  
 Polygon={E3, E4, G2, G5} Lines={F23, F24, F25, F26, E1, G1}  
 Polygon={E3, E6, G2, G4} Lines={F23, F24, F25, F26, E1, G1}  
 Polygon={E3, E6, G2, G5} Lines={F23, F24, F25, F26, E1, G1}

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 Polygon={E3, F13, F16, F24, F35} Lines={F26, F36, F46, F56, E1, G1}  
 Polygon={E4, F23, F24, F45, G2} Lines={F26, F36, F46, F56, E1, G1}

Polygon={E4, F23, F34, F46, G3} Lines={F25, F35, F45, F56, E1, G1}  
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Polygon={E5, F25, F45, G2, G5} Lines={F23, F34, F35, F36, E1, G1}  
Polygon={E6, F13, F36, F46, G3} Lines={F23, F24, F25, F26, E1, G1}  
Polygon={E6, F16, F56, G1, G6} Lines={F13, F34, F35, F36, E2, G2}  
Polygon={F12, F14, F25, F36, F46} Lines={F15, F35, F45, F56, E2, G2}  
Polygon={F12, F14, F26, F35, F56} Lines={F24, F34, F45, F46, E1, G1}  
Polygon={F12, F15, F26, F34, F46} Lines={F25, F35, F45, F56, E1, G1}  
Polygon={F13, F14, F26, F35, F46} Lines={F15, F25, F45, F56, E3, G3}  
Polygon={F14, F23, F26, F35, F45} Lines={F16, F36, F46, F56, E2, G2}  
Polygon={F14, F25, F36, F45, G4} Lines={F16, F26, F46, F56, E3, G3}  
Polygon={F15, F26, F34, F56, G6} Lines={F14, F24, F45, F46, E3, G3}  
Polygon={F16, F23, F24, F35, F56} Lines={F14, F34, F45, F46, E2, G2}  
Polygon={F16, F24, F35, F45, G5} Lines={F26, F36, F46, F56, E1, G1}